1. Motivations

Disk galaxies are complex and chaotic systems where external as well as internal perturbations drive secular evolution. Several feedback mechanisms, such as large-scale gravitational instabilities coupled to energy dissipation, make them essentially global non-linear systems in which the classical assumptions of symmetries and time-independence leading to use integrals of motion such as energy and angular momentum constraining stellar motion must be considered with caution, and are likely to have a limited time validity.

This caution to assume integrals of motion for individual particles is even more relevant if disk galaxies are in a state of marginal gravitational stability where the Safronov-Toomre stability criterion \( Q_* \) is close to critical, because then the system is globally strongly reactive to any perturbations, and stars then participate to collective gravitational effects such as spiral arms. The propensity of disk galaxies to develop small scale or grand design spiral arms is a clear indicator of global critical state.

We wish to characterize the particle diffusing behaviour observed in fully live \( N \)-body disk simulations by a standard spatial diffusion process, instead of the more traditional phase space or pure velocity space diffusion. Indeed spatial diffusion is closer to observables (such as the spatial distribution and age of stellar populations at the present time together with their spatial velocity moments) than phase space diffusion.

The concept of diffusion in a determinist dynamical systems makes sense only if the system is sufficiently chaotic at a microscopic level, which is the case of \( N \)-body systems, \( N \to 2 \), and in which the individual motion of particles appears to possess a stochastic component.

The idea here is to model in \( N \)-body simulations the apparent stochastic radial component of orbits by a spatial diffusion process following the classical Fourier’s law. In polar coordinates Fourier’s law for a distribution \( F(R,t) \) reads,

$$ \frac{\partial F}{\partial R} = \frac{1}{R} \frac{\partial}{\partial R} \left( \frac{R}{D} \frac{\partial F}{\partial R} \right) $$

In simulations the radial diffusion is characterized in radius and time by fitting the diffusion coefficient \( D(R,t) \) matching Fourier’s law coefficient. If \( D \) is approximated as a constant over a short time interval, the local elementary solution to previous equation reads

$$ F(R,t) = \frac{R_0^2}{2Dt} \exp \left[ -\frac{R_0^2 + R^2}{4Dt} \right] I_0 \left( \frac{R R_0}{2Dt} \right) $$

Where \( I_0 \) is a modified Bessel function. Diffusion from a delta function is a Gaussian function at \( R \gg R_0 \), but is different and skewed at \( R \ll R_0 \).

2. Models

Disks are modeled by a two-components Miyamoto-Nagai potential in addition to a pseudo-isothermal oblate dark halo. One of the Miyamoto-Nagai component represents a bulge. The models are suited to represent an approximate Milky Way galaxy.

All the components are self-gravitating.

The main parameter studied is the initial value of the \( Q_* \) parameter, explored in the range \( Q_* \in [1-5] \).

Varying other parameters has also been explored, such as the dark halo/disk mass ratio, without showing important effects.

The particles are run with the code Gadget-2 (Springel 2005) as collisionless particles.

At this stage of investigation it is important to restrict the number of physical factors and parameters to a minimum in order to study the diffusion process solely caused by dynamics. We assume that neglecting gas and star formation is acceptable over a few Gyr, which is long enough to describe dynamical diffusion occurring over shorter timescales.

Hence the maximum integration time is set to 2.5 Gyr. The disk-bulge component is modeled with \( 4 \times 10^6 \) particles and the dark halo with \( 2 \times 10^6 \).
3. Results

In all models a rotating strong bar develops and stays almost constant, with a slowly decaying pattern speed. Two extreme models, the cold model m1 with $Q_T = 1$ and the hot model m6 with $Q_T = 5$ summarize well the main observed effects.

**Cold model m1:**
- The diffusion parameter $D$ shows strong radial variation around the corotation radius, which evolves little in time.
- The slow outward migration of the diffusion peak is directly related to the slowly decaying pattern speed of the bar.
- Most of the stars ending at a radius $R = 8 \pm 0.1$ kpc (which could represent the Solar neighbourhood) have been spread over a several kpc broad region most of the past Gyr.

**Hot model m6:**
- The diffusion parameter $D$ presents no particular signature around corotation, but a strong diffusion in the outskirts where a $m=1$ mode develops. The overall diffusion is smaller than in the cold model.
- As a consequence most of the stars ending at a radius $R = 8 \pm 0.1$ kpc have come from a much narrower region than in the cold model.

4. Conclusions

- The amount of radial diffusion and migration is strongly dependent in radius and time on the Safronov-Toomre parameter $Q_T$. A cold disk produces more diffusion than a hot disk, which illustrates the importance of global marginal states for boosting collective effects and invalidating the good conservation of energy and angular momentum of individual particles.
- The bar plays a crucial role in cold models for producing highly variable diffusion in radius in regions around corotation and even further. This is relevant for the Milky Way, by now well confirmed to be a barred galaxy, and the Solar neighbourhood.
- Future stellar population models should include such diffusion effects, and include the bar dynamics as well as the degree of global gravitational stability of the disk.
- However predictions for time 3-12 Gyr in the past demand to have under control more physics than pure dynamics, such as gas and star formation.